

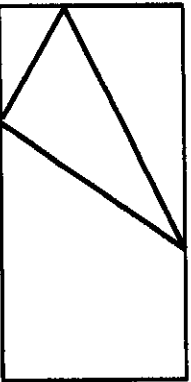
Calculus Application Problem - Keep on Folding!

The problem we are going to solve is:

Given a sheet of paper 8.5" by 11", fold the top left corner down to a point on the bottom edge. Where should you place this corner to maximize the area of the triangle formed in the bottom left corner?

Remember, the area of a triangle can be found by the geometry formula: $\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$.

This formula tells us that the area is a function of the x and the y of the triangle. Let's do some measuring (and calculating!) and determine the area for some triangles. Take your 8.5" by 11" piece of paper and (holding it sideways) mark the bottom edge in 1" units and the left side of the paper in 0.5" units. (You only need to mark the bottom edge up to 8".)



Fold the top left corner down to the point on the bottom edge that measures 1". Estimate the height of the triangle to the nearest 0.1". Now calculate the area of the triangle. We are now going to let the base of the triangle grow longer.

What will happen to the height of the triangle?

What do you think will happen to the area of the triangle?

Let's see if the above conjectures are correct. Fold the top left corner down to the indicated point on the bottom edge of the piece of paper. Estimate the height of each triangle (to the nearest 0.1 in). Then calculate the area of the triangle. Use the chart below to show your results.

BASE	HEIGHT	AREA
1		
2		
3		
4		
5		
6		
7		
8		

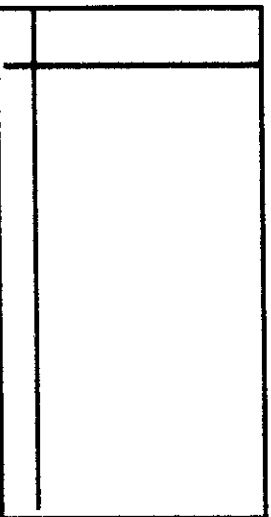
From your chart where does it appear that the maximum value of the area of the triangle occurs? The maximum area of the triangle is . It occurs when the base is and the height is .

Solutions to the Problem:

We want to determine the "exact value" of the base that will maximize the area of the triangle. We will do this two ways. First, a numerical/graphical solution (which does not require calculus), and then an analytical solution (which will require our calculus).

1. The Numerical/Graphical Solution

Let's look at the BASE, HEIGHT, and AREA data in graphical form. Put the values of the BASE in L1, the values of the HEIGHT in L2, and the values of the AREA in L3. We will first find a relationship between the BASE and the HEIGHT. Create a scatterplot of the BASE vs. HEIGHT (or L1 vs. L2). Set up your window to get a good view of your data points and plot the data. Draw a diagram of the data points below and indicate the window you used on your calculator.



[Xmin,Xmax]: _____

[Y min,Y max]: _____

What type of function do you think your data matches the closest? (It's not linear!) _____.

Using the regression option on your calculator, see if your guess was correct. Graph your regression equation.

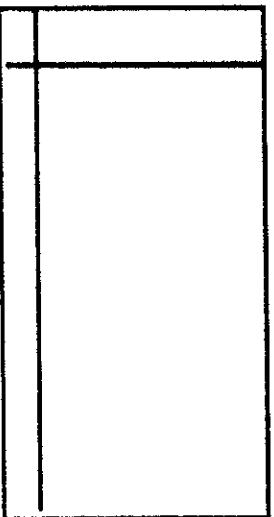
How does it fit? _____ Write the results of the regression equation below. (Round values to 2 decimal places.) Do not write the equation in terms of x and y , but in terms of B (BASE) and H (HEIGHT).

We are now ready to write our equation for the AREA of the triangle. Using the formula for the area of a triangle, the area A of the triangle (in terms of the base B) is:

$$A = \underline{\hspace{2cm}}$$

The independent variable is _____ and the dependent variable is _____.

Before we graph this function, let's see the scatterplot of the relationship between the BASE (L1) of the triangle and the AREA (L3) of the triangle. Change the set up of the scatterplot, graph it, and show your graph below. Then, graph your area function to see how it fits.



Remember, we wanted to find where to fold the paper to maximize the area of the triangle. Use the graph of your "area function" to answer this question. The maximum AREA of the triangle is _____ square inches. It occurs when the BASE of the triangle is _____ inches, and the HEIGHT of the triangle is _____ inches.

2. The Analytical Solution

In order to analytically find a function for the AREA A in terms of the BASE, again we first need to write a function for the HEIGHT H of the triangle in terms of the BASE B .

Draw a diagram in the space below of the paper with the top left corner folded down (similar to the figure on the first page). Label the base of the triangle B . Label the height of the triangle H . We need to write an expression for the length of the hypotenuse of the triangle. Your answer should be in terms of H , not B . (Hint: Unfold your paper to see how the hypotenuse relates to the height!)

The hypotenuse, in terms of the height H , is _____

Use the Pythagorean Theorem and write an equation which shows the relationship between these three sides.

$$\underline{\hspace{2cm}} = \underline{\hspace{2cm}} + \underline{\hspace{2cm}}$$

Using algebra, solve this equation for H . (It's not as hard as it looks!)

Write all decimals in the equations as fractions. $H = \underline{\hspace{2cm}}$

Compare this equation with the result of your quadratic regression equation on page 2. Although they may not appear similar, if you look closely at the coefficients you will see that they are almost the same (if you did your measurements accurately).

Using this form of the HEIGHT equation, rewrite the AREA A function in terms of the BASE B . Keep your coefficients in fraction form and distribute where appropriate to simplify the form of the function.

$$A = \underline{\hspace{2cm}}$$

Sketch this function with the AREA vs. BASE scatterplot to see how this fits. (Of course it should fit well, since it is just another form of the other function.)

It's now time for our calculus! Explain how we can use the AREA function to analytically (not graphically) determine the value of the BASE that will maximize the AREA of the triangle.

Perform this calculus (and algebra) to determine the BASE that will maximize the AREA of the triangle. Compare your answer with the answer you obtained from the graphical solution.